CURRICULUM FOR MATHEMATICS IN UNDER GRADUATE DEGREE PROGRAMME

CBCS SYLLABUS SCHEDULE 2016 - 2017
SEMESTER - VI

By

Chairperson

Board of Studies

Department of Mathematics

Kakatiya University, Warangal.

Kakatiya University
B.Sc. Mathematics, VI Semester
Skill Enhancement Course - IV
B.Sc., III Year, VI Semester
Quantitative Aptitude Test

Credits: 2 Theory: 2 hours/week **Marks - 50**

Unit I : Arithematical Ability

1.1 Arithmetical Ability: Ratio and Proportion
1.2 Arithmetical Ability: Time and Work, Time and Distance
1.3 Arithmetical Ability: Simple Interest, Compound Interest
1.4 Arithmetical Ability: Stocks and Shares

Unit II : Data Interpretation

2.1 Data Interpretation: Tabulation
2.2 Data Interpretation: Bar Graphs
2.3 Data Interpretation: Pie Charts
2.4 Data Interpretation: Line Graphs
TEXT: *Quantitative Aptitude* by Dr.R.S.Aggarwal

Kakatiya University
B.Sc. Mathematics, VI Semester
NUMERICAL ANALYSIS

DSC-1F
BS:603

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Students will be made to understand some methods of numerical analysis.

Outcome: Students realize the importance of the subject in solving some problems of algebra and calculus.

UNIT-I

Solutions of Equations in One Variable : The Bisection Method - Fixed-Point Iteration - Newtons Method and Its Extensions - Error Analysis for Iterative Methods - Accelerating Convergence - Zeros of Polynomials and Mullers Method - Survey of Methods and Software.

UNIT-II

Interpolation and Polynomial Approximation: Interpolation and the Lagrange Polynomial - Data Approximation and Nevilles Method - Divided Differences.

UNIT-III

Hermite Interpolation - Cubic Spline Interpolation. Numerical Differentiation and Integration: Numerical Differentiation - Richardsons Extrapolation

UNIT-IV

Elements of Numerical Integration- Composite Numerical Integration - Romberg Integration - Adaptive Quadrature Methods - Gaussian Quadrature.

TEXT: Richard L. Burden and J. Douglas Faires,*Numerical Analysis (9e)*
References

- M. K. Jain, S. R. K. Iyengar and R. K. Jain, *Numerical Methods for Scientific and Engineering computation*
- B. Bradie, *A Friendly introduction to Numerical Analysis*

UNIT-I

(1) Use the Bisection method to find P_3 for $f(x) = \sqrt{x} - cosx$ on [0,1].

(2) Let $f(x) = 3(x+1)(x-1/2)(x-1)$. Use the Bisection method on the following intervals to find P_3.
(a) [-2,1.5]
(b) [-1.25,2.5]

(3) Use the Bisection method to find solutions accurate with in 10^{-5} for the following problems.
(a) $x - 2^{-x} = 0$ for $0 \leq x \leq 1$
(b) $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$
(c) $2xcos(2x) - (x+1)^2 = 0$ for $-3 \leq x \leq -2$ and$-1 \leq x \leq 0$.

(4) Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$.
(a) $g_1(x) = (3 + x - 2x^2)^{1/4}$
(b) $g_2(x) = (\frac{x+3-x^4}{2})^{\frac{1}{2}}$

(5) Use a fixed-point iteration method to determine a solution accurate to with in 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1,2]. Use $p_0 = 1$.

(6) Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on [1,2] .Use $p_0 = 1$.

(7) Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within 10^{-4}.

(8) The equation $x^2 - 10cosx = 0$ has two solutions,± 1.3793646. Use Newton's method to approximate the solutions to within 10^{-5} with the following values of P_0.
(a) $P_0 = -100$
(b) $P_0 = -50$
(c) $P_0 = -25$
(d) $P_0 = 25$
(e) $P_0 = 50$
(f) $P_0 = 100$

(9) The equation $4x^2 - e^x - e^{-x} = 0$ has two positive solutions x_1 and x_2.Use Newton's method to approximate the solution to within 10^{-5} with the following values of p_0.
(a) $P_0 = -10$ (b) $P_0 = -5$ (c) $P_0 = -3$
(d) $P_0 = -1$ (e) $P_0 = 0$ (f) $P_0 = 1$
(g) $P_0 = 3$ (h) $P_0 = 5$ (i) $P_0 = 10$

(10) Use each of the following methods to find a solution in $[0.1, 1]$ accurate to within 10^{-4} for $600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0$
(a) Bisection method
(b) Newton method
(c) Secant method
(d) Method of False position
(e) Muller's method

UNIT-II

(11) For the given function $f(x)$, let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct interpolation polynomial of degree at most one and at most two to approximate f(0.45),and find the absolute error
(a) $f(x) = cosx$ (b) $f(x) = ln(x+1)$

(12) For the given function $f(x)$, let $x_0 = 1$, $x_1 = 1.25$ and $x_2 = 1.6$. Construct interpolation polynomial degree at most one and at most two to approximate f(1.4), and find the absolute error.
(a) $f(x) = sin\pi x$ (b) $f(x) = log(3x-1)$

(13) Let $P_3(x)$ be the interpolating polynomials for the data $(0,0)$,$(0.5,y)$,$(1,3)$ and $(2,2)$. The coefficient of x^3 in $P_3(x)$ is 6. Find y

(14) Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$			
$x_1 = 0.25$	$P_1 = 2$	$P_{0,1} = 2.6$		
$x_2 = 0.5$	P_2	$P_{1,2}$	$P_{0,1,2}$	
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$

Determine $P_2 = f(0.5)$.

(15) Neville's method is used to approximate $f(0.5)$, giving the following table.

$x_0 = 0$	$P_0 = 0$		
$x_1 = 0.4$	$P_1 = 2.8$	$P_{0,1} = 3.5$	
$x_2 = 0.7$	P_2	$P_{1,2}$	$P_{0,1,2} = \frac{27}{7}$

Determine $P_2 = f(0.7)$.

(16) Neville's Algorithm is used to approximate $f(0)$ using $f(-2)$,$f(-1)$,$f(1)$ and $f(2)$. Suppose $f(-1)$ was overstated by 2 and $f(1)$ was understated by 3. Determine the error in the original calculation of the value of the interpolating polynomial to approximate $f(0)$.

(17) Compute the divided difference table for the data

x	1.0	1.3	1.6	1.9	2.2
$f(x)$	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

(18) Use the Newton forward-difference formula to construct interpolating polynomaials of degree one,two,and three for the following data. Approximate the specified value using each of the polynomials.
(a) $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$
(b) $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$

(19) Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.
(a) $f(0.43)$ if $f(0) = 1$,$f(0.25) = 1.64872$,$f(0.5) = 2.71828$,$f(0.75) = 4.48169$
(b) $f(0.25)$ if $f(-1) = 0.86199480$, $f(-0.5) = 0.95802009$, $f(0) = 1.0986123$, $f(0.5) = 1.2943767$

(20) Use Stirling's formula to approximate $f(0.43)$ for the following data

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.0000	1.22140	1.49182	1.82212	2.22554

UNIT-III

(21) Use the Hermite Polynomial to find an approximation of $f(1.5)$ for the following data

k	x_k	$f(x_k)$	$f'(x_k)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.56989959
2	1.9	0.2818186	-0.5811571

(22) A car travelling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

Use Hermites polynomial to predict the position of the car and its speed when $t = 10 second$

(23) Use the following values and five - digit - rounding arithematic to construct the Hermite interpolating polynomial to approximate $sin(0.34)$

x	sin$x$	$D_x sinx = cosx$
0.30	0.29552	0.95534
0.32	0.31457	0.94924
0.35	0.34290	0.93937

(24) Determine the natural cubic spline S that interpolates the data $f(0) = 0$,$f(1) = 1$,and $f(2) = 2$.

(25) Determine the clamped cubic spline S that interpolates the data $f(0) = 0$,$f(1) = 1$,$f(2) = 2$, and satisfies $s'(0) = s'(2) = 1$.

(26) Use the forward-difference formula and backward-difference formula to determine each missing entry in the following tables.

(a)

x	$f(x)$	$f'(x)$
0.5	0.4794	
0.6	0.5646	
0.7	0.6442	

(b)

x	$f(x)$	$f'(x)$
0.0	0.0000	
0.2	0.74140	
0.4	1.3718	

(27) Consider the following table of data

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.9798652	0.9177710	0.808038	0.6386093	0.3843735

Use all the appropriate formulas given in this section to approxmate $f'(0.4)$ and $f''(0.4)$.

(28) Derive a method for approximating $f'''(x_0)$ whose error term is of order h^2 by expanding the function f in a fourth Taylor polynomial about x_0 and evaluating at $x_0 \pm h$ and $x_0 \pm 2h$.

(29) The forward-difference formula can be expressed as

$f'(x_0) = \frac{1}{h}[f(x_0+h) - f(x_0)] - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3)$. Use extrapolation to derive $O(h^3)$ formula for $f^{'}(x_0)$

(30) Show that $lim_{h \longrightarrow 0}(\frac{2+h}{2-h})^{\frac{1}{h}} = e$

UNIT-IV

(31) Approximation the following integrals using the Trapezoidal rule.
(a) $\int_{0.5}^{1} x^4 dx$
(b) $\int_{0}^{0.5} \frac{2}{x-4} dx$
(c) $\int_{1}^{1.5} x^2 lnx dx$
(d) $\int_{0}^{1} x^2 e^{-x} dx$

(32) Approximate the following integral using Trapezoidal Rule
(a) $\int_{-0.25}^{0.25} (cosx)^2 dx$
(b) $\int_{-0.5}^{0} xln(x+1) dx$

(33) The Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 5, and the midpoint rule gives the value 4. What value does Simpson's rule give?

(34) The quadrature formula $\int_0^2 f(x)dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0,c_1, and c_2.

(35) Find the constants c_0, c_1 and x_1 so that quadrature formula $\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$ has the highest possible degree of precision.

(36) Use the composite Trapezoidal Rule with the indicated values of n to approximate the following integrals
(a) $\int_1^2 xlnx dx$, n=4
(b) $\int_{-2}^{2} x^3 e^x dx$, n=4.

(37) Suppose that $f(0) = 1, f(0.5) = 2.5, f(1) = 2$ and $f(0.25) = f(0.75) = \infty$. Find ∞ if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)dx$

(38) Romberg integration is used to approximate $\int_2^3 f(x)dx$.
If $f(2) = 0.51342$,$f(3) = 0.36788$ $R_{31} = 0.43687$, $R_{33} = 0.43662$, find $f(2.5)$

(39) Use Romberg integration to compute $R_{3,3}$ for the following integrals.
(a) $\int_{1}^{1.5} x^2 lnx dx$
(b) $\int_{0}^{1} x^2 e^{-x} dx$

(40) Use Romberg integration to compute $R_{3,3}$ for the following integrals.
(a) $\int_{-1}^{1} (cosx)^2 dx$
(b) $\int_{-0.75}^{0.75} xln(x+1) dx$

Kakatiya University
B.Sc. Mathematics, VI Semester
COMPLEX ANALYSIS

DSE-1F/A
BS:606

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Analytic Functions, contour integration and calculus of residues will be introduced to the students.

Outcome: Students realize calculus of residues is one of the power tools in solvng some problems, like improper and definite integrals, effortlessly.

UNIT-I

Regions in the Complex Plane - Analytic Functions - Functions of a Complex Variable - Mappings - Mappings by the Exponential Function - Limits - Theorems on Limits - Limits Involving the Point at Infinity - Continuity - Derivatives - Differentiation Formulas - Cauchy-Riemann Equations - Sufficient Conditions for Differentiability - Polar Coordinates-Harmonic Functions.

UNIT-II

Elementary Functions: The Exponential Function - The Logarithmic Function - Branches and Derivatives of Logarithms - Some Identities Involving Logarithms Complex Exponents - Trigonometric Functions - Hyperbolic Functions.

UNIT-III

Integrals: Derivatives of Functions w(t) - Definite Integrals of Functions w(t) - Contours - Contour Integrals - Some Examples - Examples with Branch Cuts - Upper Bounds for Moduli of Contour Integrals -Antiderivatives.

UNIT-IV

Cauchy-Goursat Theorem - Proof of the Theorem - Simply Connected Domains - Multiply Connected Domains - Cauchy Integral Formula - An Extension of the Cauchy Integral Formula - Some Consequences of the Extension - Liouville's Theorem and the Fundamental Theorem of Algebra- Maximum Modulus Principle.

TEXT: James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications* (8e)
References:

- Joseph Bak and Donald J Newman, *Complex analysis*
- Lars V Ahlfors , *Complex Analysis*
- S.Lang, *Complex Analysis*
- B Choudary, *The Elements Complex Analysis*

UNIT-I

(1) Sketch the following set and determine which are domains (a) $|z-2+i| \leq 1$
(b) $|2z+3| > 4$
(c) $Imz > 1$
(d) $Imz = 1$.

(2) Sketch the region onto which the sector $r \leq 1, 0 \leq \theta \leq \frac{\pi}{4}$ is mapped by the transformation

(a) $w = z^2$

(b) $w = z^3$

(c) $w = z^4$

(3) Find all roots of the equation
(a) $sinhz = i$ (b) $coshz = \frac{1}{2}$

(4) Find all values of z such that
(a) $e^z = -2$; (b) $e^z = 1 + \sqrt{3}i$; (c) $exp(2z-1) = 1$.

(5) Show that
$lim_{z \to z_0} f(z)g(z)$ if $lim_{z \to z_0} f(z) = 0$
and if there exists a positive number M such that $|g(z)| \leq M$ for all z in some neighborhood of z_0.

(6) Show that $f'(z)$ does not exist at any point if
(a) $f(z) = \overline{z}$ (b)$f(z) = z - \overline{z}$
(c)$f(z) = 2x + ixy^2$ (d)$f(z) = e^x e^{-iy}$

(7) Verify that each of these functions is entire
(a)$f(z) = 3x + y + i(3y - x)$ (b)$f(z) = sinxcoshy + icosxsinhy$
(c)$f(z) = e^{-y}sinx - ie^{-y}cosx$ (d)$f(z) = (z^2-2)e^{-x}e^{-iy}$.

(8) State why a composition of two entire functions is entire. Also, state why any linear combination $c_1 f_1(z) + c_2 f_2(z)$ of two entire functions, where c_1 and c_2 are complex constants, is entire.

(9) Show that $u(x,y)$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ when
(a) $u(x,y) = 2x(1-y)$ (b)$u(x,y) = 2x - x^3 + 3xy^2$
(c)$u(x,y) = sinhxsiny$ (d) $u(x,y) = \frac{y}{x^2+y^2}$

(10) Show that if v and V are harmonic conjugates of $u(x,y)$ in a domain D, then $v(x,y)$ and $V(x,y)$ can differ at most by an additive constant.

UNIT-II

(11) Show that $exp(z + \pi i) = -exp(z)$

(12) Find all values of z such that $e^z = -2$

(13) Show that $exp\overline{z} = \overline{expz}$ $\forall z$ and $exp(\overline{iz}) = \overline{exp(iz)}$

(14) Show that the function $exp\overline{z}$ is not analytic any where

(15) Show that $cos(i\overline{z}) = \overline{cos(iz)}\ \forall z$
$sin(i\overline{z}) = \overline{sin(iz)}$ if and only if $z = n\pi i\ (n = 0, \pm1, \pm2....)$

(16) Show that neither $sin\overline{z}$ nor $cos\overline{z}$ is an analytic function of z any where

(17) Show that $sin^{-1}(-i) = n\pi + i(-1)^{n+1}cos(1+\sqrt{2})\ (n = 0, \pm1, \pm2....)$

(18) Show that $cos(-ei) = 1 - \frac{\pi}{2}i$

(19) Find all the roots of the equation $coshz = \frac{1}{2}$

(20) Find all the root of the equation $sinhz = i$

UNIT-III

(21) Evaluate $\int_C f(z)dz$
where $f(z) = \frac{(z+2)}{z}$ and C is

(a) the semicircle $z = 2e^{i\theta}(0 \le \theta \le \pi)$
(b) the semicircle $z = 2e^{i\theta}(\pi \le \theta \le 2\pi)$
(c) the circle $z = 2e^{i\theta}(0 \le \theta \le 2\pi)$

(22) $f(z)$ is defined by the means of the equations $f(z) = \begin{cases} 1 & when\, y < 0 \\ \text{4y} & when\, y > 0 \end{cases}$ and C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$, then find $\int_C f(z)dz$.

(23) Let C denote the line segment from $z = i$ to $z = 1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that
$|\int_C \frac{dz}{z^4}| \le 4\sqrt{2}$
without evaluating the integral.

(24) Let C_R denote the upper half of the circle $|z| = R(R > 2)$, taken in the counter clockwise direction. Show that
$|\int_{C_R} \frac{2z^2-1}{z^4+5z^2+4}dz| \le \frac{\pi R(2R^2+1)}{(R^2-1)(R^2-4)}$.
Then , by dividing the numerator on the right here by R^4 , show that the value of the integral tends to zero as R tends to infinity.

(25) By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:
(a)$\int_i^{i/2} e^{\pi z}dz$ (b)$\int_0^{\pi+2i} cos(\frac{z}{2})dz$ (c)$\int_1^3 (z-2)^3 dz$

(26) Use an antiderivative to show that for every contour C extending from a point z_1 to a point z_2,
$\int_C z^n dz = \frac{1}{n+1}(z_2^{n+1} - z_1^{n+1})$ $(n = 0, 1, 2, ...)$

(27) Let C_0 and C denote the circle $z = z_0 + Re^{i\theta}(-\pi \le \theta \le \pi)$ and $z = Re^{i\theta}(-\pi \le \theta \le \pi)$ respectively.
(a) Use these parametric representations to show that
$\int_{C_0} f(z - z_0)dz = \int_C f(z)dz$

(28) Evaluate the integral $\int_C z^m z^{-n} dz$
where m and n are integers and C is the unit circle $|\ z\ |= 1$ taken counterclockwise.

(29) $f(z) = 1$ and C is an arbitary contour from any fixed point z_1 to any fixed point z_2 in the z plane .Evaluate
$\int_C f(z)dz$

(30) $f(z) = \pi exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1+i$ and i the orientation of C being in the counterclockwise direction .Evaluate
$\int_c f(z)dz$

UNIT-IV

(31) Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals.
$a.\ \int_C \frac{e^{-z}}{z-(\frac{\pi i}{2})}dz$
$b.\ \int_C \frac{cosz}{z(z^2+8)}dz$
$c.\ \int_C \frac{z}{2z+1}dz$

(32) Find the value of the integral $g(z)$ around the circle$\mid z - i \mid= 2$ in the positive sense when
a.$g(z) = \frac{1}{z^2+4}$
b.$g(z) = \frac{1}{(z^2+4)^2}$

(33) C be the circle $\mid z \mid= 3$ described in the positive sense. Show that if
$g(z) = \int_C \frac{2s^2-s-2}{s-z}dz,\ (\mid z \mid\neq 3)$
then $g(2) = 8\pi i$. What is the value of $g(z)$ when $\mid z \mid> 3$?

(34) Let C be any simple closed contour ,described in the positive sense in z plane ,and write
$g(z) = \int_C \frac{s^3+2s}{(s-z)^3}dz$
Show that $g(z) = 6\pi iz$ when z is inside C and that $g(z) = 0$ when z is outside.

(35) Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then
$\int_C \frac{f'(z)}{z-z_0}dz = \int_C \frac{f(z)}{(z-z_0)^2}dz$

(36) Let C be the unit circle $z = e^{i\theta}$ $(-\pi \leq \theta \leq \pi)$. First show that for any real constant a
$\int_C \frac{e^{az}}{z}dz = 2\pi i$
Then write this integral in terms of θ to derive the integration formula
$\int_0^\pi e^{acos\theta}cos(asin\theta)d\theta = \pi$

(37) suppose that $f(z)$ is entire and that the harmonic function $u(x, y) = Re|f(z)|$ has an upper bound u_0; that is $u(x, y) \leq u_0$ in the xyplane. Show that $u(x, y)$ must be constant throughtout the plane.

(38) Let a function f be continuous on a closed bounded region R, and let it be analytic and not constant throughout the interior of R. Assuming that $f(z) \neq 0$ anywhere in R. Prove that $|f(z)|$ has a minimum value m in R which occur on the boundary of R and never in the interior. Do this by applying the corresponding result for maximum values to the function $g(z) = \frac{1}{f(z)}$

(39) Let the function $f(z) = u(x, y) + iv(x, y)$ be continuous on a closed bounded region R, and suppose that it is analytic and non constant in the interior of R. Show that the component function $v(x, y)$ has maximum and minimum values in R which are reached on the boundary of R and never in the interior, where it is harmonic

(40) Let f be the function $f(z) = e^z$ and R the rectangular region $0 \leq x \leq 1$, $0 \leq y \leq \pi$. Find points in R where the component function $u(x, y) = Re[f(z)]$ reaches its maximum and minimum values

Kakatiya University
B.Sc. Mathematics, VI Semester
VECTOR CALCULUS

DSE-1F/B
BS:606

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Concepts like gradient, divergence, curl and their physical relevance will be taught.

Outcome: Students realize the way vector calculus is used to addresses some of the problems of physics.

UNIT- I

Line Integrals: Introductory Example : Work done against a Force-Evaluation of Line IntegralsConservative Vector Fields

UNIT- II

Surface Integrals: Introductory Example : Flow Through a PipeEvaluation of Surface Integrals. Volume Integrals: Evaluation of Volume integrals

UNIT- III

Gradient, Divergence and Curl: Partial differentiation and Taylor series in more than one variable-Gradient of a scalar field-Gradients, conservative fields and potentials-Physical applications of the gradient.

UNIT- IV

Divergence of a vector field -Physical interpretation of divergence-Laplacian of a scalar field- Curl of a vector field-Physical interpretation of curl-Relation between curl and rotation-Curl and conservative vector fields.
TEXT: P.C. Matthews, *Vector Calculus*
References:

- G.B. Thomas and R.L. Finney, *Calculus*
- H. Anton, I. Bivens and S. Davis ; *Calculus*
- Smith and Minton, *Calculus*

UNIT-I

(1) Evaluate the line integral $\int_c F \times dr$, where F is the vector field $(y, x, 0)$ and C is the curve $y = sinx$, $z = 0$, between $x = 0$ and $x = \pi$.

(2) Evaluate the line integral $\int_c x + y^2 dr$, where c is the parabola $y = x^2$ in the plane $z = 0$ connecting the points $(0, 0, 0)$ and $(1, 1, 0)$.

(3) Evaluate the line integral $\int_c f.dr$, where F=$(5z^2, 2x, x + 2y)$ and the curve C is given by $x = t, y = t^2, z = t^2$, $0 \leq t \leq 1$

(4) Find the line integral of the vector field $u = (y^2, x, z)$ along the curve given by $z = y = e^x$ from $x = 0$ and $x = 1$.

(5) Evaluate the line integral of the vector field $u = (xy, z^2, x)$ along the curve given by $x = 1+t, y = 0, z = t^2$,$0 \leq t \leq 3$.

(6) Find the line integral of $F = (y, -x, 0)$ along the curve consisting of the two straight line segments $y = 1$, $0 \leq x \leq 1$.

(7) Find the circulation of the vector $F = (y, -x, 0)$ around the unit circle $x^2 + y^2 = 1$, $z = 0$, taken in anticlockwise direction.

(8) Find the line integral $\oint r * dr$,where the curve C is the ellipse $x^2/a^2 + y^2/b^2 = 1$ taken in an anticlockwise direction.What do you notice about the magnitude of the answer?

(9) By considering the line integral of $F = (y, x^2 - x, 0)$ around the square in the x,y plane connecting the four points $(0, 0)$,$(1, 0)$,$(1, 1)$ and $(0, 1)$,show that F cannot be a conservative vector field.

(10) Evaluate the line integral of the vector field $u = (xy, z^2, x)$ along the curve given by $x = 1+t, y = 0, z = t^2$,$0 \leq t \leq 3$.

UNIT-II

(11) Evaluate the surface integral of $u = (y, x^2, z^2)$,over the surface S,where S is the triangular surface on $x = 0$ with $y \geq 0$,$z \geq 0$,$y + x \leq 1$,with the normal n directed in the positive x direction

(12) Find the surface integral of $u = r$ over the part of the paraboloid $z = 1 - x^2 - y^2$ with $z > 0$, with the normal pointing upwards.

(13) If S is the entire x,y plane,evaluate the integral $I = \int_s e^{-x^2-y^2} ds$, by transforming the integral into polar coordinates.

(14) A cube $0 \leq x, y, z \leq 1$ has a variable density given by $\rho = 1 + x + y + z$. what is the total mass of the cube?

(15) Find the volume of the tetrahedron with vertices (0, 0, 0), (a, 0, 0), (0, b, 0) (0, 0, c).

(16) Evaluate the surface integral of $\mathbf{u} = (xy, x, x + y)$ over the surface S defined by $z = 0$ with $0 \leq x \leq 1$, $0 \leq y \leq 2$, with the normal $\mathbf{n}$ directed in the positive z direction.

(17) The surface S is defined to be that part of the plane $z = 0$ lying between the curve $y = x^2$ and $x = y^2$. Find the surface integral of $\mathbf{u.n}$ over S where $u = (z, xy, x^2)$ and $\mathbf{n} = (0, 0, 1)$.

(18) Find the surface integral of $\mathbf{u}.\mathbf{n}$ over S where S is the part of the surface $z = x + y^2$ with $z < 0$ and$x > -1$, u is the vector field $\mathbf{u} = (2y + x, -1, 0)$ and $\mathbf{n}$ has a negative z component.

(19) Find the volume integral of the scalar field $\phi = x^2 + y^2 + z^2$ over the region V specified by $0 \leq x \leq 1, 1 \leq y \leq 2, 0 \leq z \leq 3$.

(20) Find the volume of the section of the cylinder $x^2 + y^2 = 1$ that lies between the planes $z = x + 1$ and $z = -x - 1$.

(21) Find the unit normal $\mathbf{n}$ to the surface $x^2 + y^2 - z = 0$ at the point $(1, 1, 2)$.

(22) find the gradient of the scalar field $f = xyz$ and evaluate it at the point $(1, 2, 3)$. Hence find the diraction derivative of f at this point in the direction of the vector $(1, 1, 0)$.

UNIT-III

(23) Find the divergence of the vector field **u=r**.

(24) The vector field $\mathbf{u}$ is defined by $\mathbf{u} = (xy, z + x, y)$. Calculate $\bigtriangledown \times u$ and find the point where $\bigtriangledown \times u = 0$.

(25) Find the gradient $\bigtriangledown\phi$ and the Laplacian $\bigtriangledown^2\phi$ for the scalar field $\phi = x^2 + xy + yz^2$.

(26) Find the gradient and the Laplacian of $\phi = \sin(kx)\sin(ly)e^{\sqrt{k^2+l^2}z}$.

(27) Find the unit normal to the surface $xy^2 + 2yz = 4$ at the point (-2, 2, 3).

(28) For $\phi(x, y, z) = x^2 + y^2 + z^2 + xy - 3x$, find $\bigtriangledown\phi$ and find the minimum value of ϕ.

(29) Find the equation of the plane which is tangent to the surface $x^2 + y^2 - 2z^3 = 0$ at the point (1, 1, 1).

(30) Prove that $\nabla^2(\frac{1}{r}) = 0$

UNIT-IV

(31) Find both the divergence and the curl of the vector fields
(a) $\mathbf{u} = (y, z, x)$;
(b) $V = (xyz, z^2, x - y)$.

(32) For what values, if any, of the constants a and b is the vector field $\mathbf{u} = (y\cos x + axz, b\sin x + z, x^2 + y)$ irrotational?

(33) (a) Show that $\mathbf{u} = (y^2z, -z^2\sin y + 2xyz, 2z\cos y + y^2x)$ is irrotational.
(b) Find the corresponding potential function.
(c) Hence find the value of the line integral of $\mathbf{u}$ along the curve
$x = \sin\frac{\pi t}{2}, y = t^2 - t, z = t^4, 0 \leq t \leq 1$.

(34) Find the divergence of the vector field $u = \overrightarrow{r}$.

(35) The vector field u is defined by $u = (xy, x+z, y)$, then calculate $\nabla \times u$ and find the points where $\nabla \times u = 0$.

(36) Show that both the divergence and the curl are linear operators.

(37) Find $\nabla . \nabla \phi$ if $\phi = 2x^3y^2z^4$.

(38) If $A = x^2yi - 2xzj + 2yzk$ then find $curl\ curl\ A$.

(39) Show that $div\ curl\ A = 0$.

(40) If $A = xz^3i - 2x^2yzj + 2yz^4k$ then find $\nabla \ \times \ A$ at the point $(1, -1, 1)$.